Study $X_c(3250)$ as a $D_0^*(2400)N$ molecular state

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We present a QCD sum rule analysis for the the newly observed resonance $X_c(3250)$ by assuming it as a $D_0^*(2400)N$ molecular state. Technically, contributions of operators up to dimension 12 are included in the operator product expansion (OPE). The numerical result is 3.18 ± 0.51 GeV for the $D_0^*(2400)N$ molecular state, which is in agreement with the experimental data of $X_c(3250)$. This supports the statement that $X_c(3250)$ could be a $D_0^*(2400)N$ molecular state. The mass for the bottom counterpart B_0^*N state is predicted to be 6.50 ± 0.49 GeV.

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I. INTRODUCTION

Very recently, BaBar Collaboration reported the measurement of the baryonic B decay $B^- \to \Sigma_c^{++}\bar{p}\pi^-\pi^-$ and observed a new structure in the $\Sigma_c^{++}\pi^-\pi^-$ invariant mass spectrum at 3.25 GeV [1]. For simplicity, one could name the new structure as $X_c(3250)$. Soon after the experimental observation, He *et al.* have suggested that $X_c(3250)$ could be a $D_0^*(2400)N$ molecular state from an effective Lagrangian calculation [2]. Theoretically, the molecular concept is well and truly not a new topic but with a history. It was put forward nearly 40 years ago in Ref. [3] and was predicted that molecular states have a rich spectroscopy in Ref. [4]. The possible deuteron-like two-meson bound states were studied in Ref. [5]. In recent years, some of “X”, “Y”, and “Z” new hadrons are ranked as possible molecular candidates. Such as, $X(3872)$ could be a $D\bar{D}^*$ molecular state [6–10]; $X(4350)$ is interpreted as a $D_s^*\bar{D}_{s0}^*$ state [11, 12]; $Y(4260)$ is proposed to be a $\chi_c\rho^0$ [13] or an $\omega\chi_{c1}$ state [14]; $Z^+(4430)$ is deciphered as a $D^*\bar{D}_1$ state [15, 16]; $Z_b(10610)$ and $Z_b(10650)$ could be $B^*\bar{B}$ and $B^*\bar{B}^*$, respectively [17, 18]. If molecular states can be completely confirmed by experiment, QCD will be further testified and then one will understand the QCD low-energy behaviors more deeply. Therefore, it is interesting to study whether the newly observed $X_c(3250)$ state could be a $D_0^*(2400)N$ molecular state. Especially, the new resonance may open a new window to study molecular states with the meson-baryon configuration.

In the real world, quarks are confined inside hadrons and the strong interaction dynamics of hadronic systems is governed by nonperturbative QCD effect completely. Many questions concerning dynamics of the quarks and gluons at large distances remain unanswered or understood only at a qualitative level. It is quite difficult to extract hadronic information quantitatively from the basic theory of QCD. The QCD sum rule method [19] is a nonperturbative formulation firmly based on the first principle of QCD, which has been successfully applied to conventional hadronic systems, i.e. mesons or baryons (for reviews see [20–23] and references therein). For multiquark states, there have appeared fruitful results from QCD sum rules these years (for a review on multiquark QCD sum rules one can see [24] and references therein). In particular for hadrons containing five quarks, some authors began to study light pentaquark states in Refs. [25]. The application of QCD sum rules to heavy pentaquark states was performed in Ref. [26] for the first time.

In this work, we devote to investigating that whether the newly observed resonance $X_c(3250)$ could be a $D_0^*(2400)N$ molecular state in the framework of QCD sum rules. As a byproduct, the mass for the bottom counterpart B_0^*N is also predicted. The rest of the paper is organized as three sections. We discuss QCD sum rules for molecular states in Sec. II utilizing similar techniques as our previous works [27]. The numerical analysis and discussions are presented in Sec. III, and masses of $D_0^*(2400)N$ and B_0^*N molecular states are extracted out. The Sec. IV includes a brief summary and outlook.

II. QCD SUM RULES FOR MOLECULAR STATES

The QCD sum rules for molecular states are constructed from the two-point correlation function

$$\Pi(q^2) = i\int d^4x e^{iq.x}\langle 0|T[j(x)\overline{j}(0)]|0\rangle. \tag{1}$$

In full theory, the interpolating current for $D_0^*(2400)$ or B_0^* meson can be found in Ref. [28], and the one for nucleon N has been listed in Ref. [29]. One can construct the $D_0^*(2400)N$ or B_0^*N molecular state current from meson-baryon type of fields

$$j \;=\; (\bar{Q}_{c'}q_{c'})(\varepsilon_{abc}q_1^{Ta}C\gamma_\mu q_2^b\gamma_\mu\gamma_5 q_3^c), \tag{2}$$

where Q is heavy quark c or b, and q_1, q_2, as well as q_3 denote light quarks u and/or d. The index T means matrix transposition, C is the charge conjugation matrix, with a, b, c and c' are color indices. One should note that meson-baryon molecules in the real world are long objects in which the quark pairs are far away from each other. The currents in this work and in most of the QCD sum rule works are local and the five field operators here act at the same space-time point. It is a limitation inherent in the QCD sum rule disposal of the hadrons since the bound states are not point particles in a rigorous manner.

Lorentz covariance implies that the two-point correlation function in Eq. (1) has the form

$$\Pi(q^2) = \Pi_1(q^2) + \not{q}\Pi_2(q^2). \tag{3}$$

According to the philosophy of QCD sum rules, the correlator is evaluated in two ways. Phenomenologically, the correlator can be expressed as a dispersion integral over a physical spectral function

$$\Pi(q^2) = \lambda_H^2\frac{\not{q}+M_H}{M_H^2-q^2} + \frac{1}{\pi}\int_{s_0}^{\infty} ds\frac{\mathrm{Im}\Pi_1^{\mathrm{phen}}(s)+\not{q}\mathrm{Im}\Pi_2^{\mathrm{phen}}(s)}{s-q^2} + \text{subtractions}, \tag{4}$$

where M_H is the mass of the hadronic resonance, and λ_H gives the coupling of the current to the hadron $\langle 0|j|H\rangle = \lambda_H$. In the OPE side, short-distance effects are taken care of by Wilson coefficients, while long-distance confinement effects are included as power corrections and parameterized in terms of vacuum expectation values of local operators, the so-called condensates. One can write the correlation function in the OPE side in terms of a dispersion relation

$$\Pi(q^2) = \int_{m_Q^2}^{\infty} ds\frac{\rho_1(s)}{s-q^2} + \Pi_1^{\mathrm{cond}}(q^2) + \not{q}\bigg\{\int_{m_Q^2}^{\infty} ds\frac{\rho_2(s)}{s-q^2} + \Pi_2^{\mathrm{cond}}(q^2)\bigg\}, \tag{5}$$

where the spectral density is given by the imaginary part of the correlation function

$$\rho_i(s) = \frac{1}{\pi}\mathrm{Im}\Pi_i^{\mathrm{OPE}}(s), \quad i=1,2. \tag{6}$$

Technically, one works at leading order in α_s and considers condensates up to dimension 12. To keep the heavy-quark mass finite, one can use the momentum-space expression for the heavy-quark propagator [28]

$$\begin{aligned}
S_Q(p) \;=\; & \frac{i}{\not{p}-m_Q} - \frac{i}{4}gt^AG^A_{\kappa\lambda}(0)\frac{1}{(p^2-m_Q^2)^2}[\sigma_{\kappa\lambda}(\not{p}+m_Q)+(\not{p}+m_Q)\sigma_{\kappa\lambda}] \\
& - \frac{i}{4}g^2t^At^BG^A_{\kappa\lambda}(0)G^B_{\mu\nu}(0)\frac{\not{p}+m_Q}{(p^2-m_Q^2)^5}[\gamma_\alpha(\not{p}+m_Q)\gamma_\beta(\not{p}+m_Q)\gamma_\mu(\not{p}+m_Q)\gamma_\nu \\
& + \gamma_\alpha(\not{p}+m_Q)\gamma_\mu(\not{p}+m_Q)\gamma_\beta(\not{p}+m_Q)\gamma_\nu + \gamma_\alpha(\not{p}+m_Q)\gamma_\mu(\not{p}+m_Q)\gamma_\nu(\not{p}+m_Q)\gamma_\beta](\not{p}+m_Q) \\
& + \frac{i}{48}g^3f^{ABC}G^A_{\gamma\delta}G^B_{\delta\varepsilon}G^C_{\varepsilon\gamma}\frac{1}{(p^2-m_Q^2)^6}(\not{p}+m_Q)[\not{p}(p^2-3m_Q^2)+2m_Q(2p^2-m_Q^2)](\not{p}+m_Q).
\end{aligned} \tag{7}$$

The light-quark part of the correlation function can be calculated in the coordinate space, with the light-quark propagator

$$\begin{aligned}
S_{ab}(x) &= \frac{i\delta_{ab}}{2\pi^2x^4}\not{x} - \frac{m_q\delta_{ab}}{4\pi^2x^2} - \frac{i}{32\pi^2x^2}t^A_{ab}gG^A_{\mu\nu}(\not{x}\sigma^{\mu\nu}+\sigma^{\mu\nu}\not{x}) - \frac{\delta_{ab}}{12}\langle\bar{q}q\rangle + \frac{i\delta_{ab}}{48}m_q\langle\bar{q}q\rangle\not{x} \\
&- \frac{x^2\delta_{ab}}{3\cdot 2^6}\langle g\bar{q}\sigma\cdot Gq\rangle + \frac{ix^2\delta_{ab}}{2^7\cdot 3^2}m_q\langle g\bar{q}\sigma\cdot Gq\rangle\not{x} - \frac{x^4\delta_{ab}}{2^{10}\cdot 3^3}\langle\bar{q}q\rangle\langle g^2G^2\rangle, \qquad (8)
\end{aligned}$$

which is then Fourier-transformed to the momentum space in D dimension. The resulting light-quark part is combined with the heavy-quark part before it is dimensionally regularized at $D=4$. Equating the two sides for $\Pi(q^2)$ and assuming quark-hadron duality yield the sum rules, from which masses of hadrons can be determined. After making a Borel transform and transferring the continuum contribution to the OPE side, the sum rules can be written as

$$\lambda_H^2 M_H e^{-M_H^2/M^2} = \int_{m_Q^2}^{s_0} ds\rho_1(s)e^{-s/M^2} + \hat{B}\Pi_1^{\text{cond}}, \qquad (9)$$

$$\lambda_H^2 e^{-M_H^2/M^2} = \int_{m_Q^2}^{s_0} ds\rho_2(s)e^{-s/M^2} + \hat{B}\Pi_2^{\text{cond}}, \qquad (10)$$

where M^2 indicates the Borel parameter. To eliminate the hadron coupling constant λ_H and extract the resonance mass M_H, one can take the derivative of Eq. (9) with respect to $1/M^2$, divide the result by itself and deal with Eq. (10) in the same way to get

$$M_H^2 = \left\{\int_{m_Q^2}^{s_0} ds\rho_1(s)se^{-s/M^2} + d/d(-\frac{1}{M^2})\hat{B}\Pi_1^{\text{cond}}(s)\right\}/\left\{\int_{m_Q^2}^{s_0} ds\rho_1(s)e^{-s/M^2} + \hat{B}\Pi_1^{\text{cond}}(s)\right\}, \qquad (11)$$

$$M_H^2 = \left\{\int_{m_Q^2}^{s_0} ds\rho_2(s)se^{-s/M^2} + d/d(-\frac{1}{M^2})\hat{B}\Pi_2^{\text{cond}}(s)\right\}/\left\{\int_{m_Q^2}^{s_0} ds\rho_2(s)e^{-s/M^2} + \hat{B}\Pi_2^{\text{cond}}(s)\right\}, \qquad (12)$$

where

$$\begin{aligned}
\rho_i(s) &= \rho_i^{\text{pert}}(s) + \rho_i^{\langle\bar{q}q\rangle}(s) + \rho_i^{\langle\bar{q}q\rangle^2}(s) + \rho_i^{\langle g\bar{q}\sigma\cdot Gq\rangle}(s) + \rho_i^{\langle g^2G^2\rangle}(s) + \rho_i^{\langle g^3G^3\rangle}(s) + \rho_i^{\langle\bar{q}q\rangle^3}(s) + \rho_i^{\langle\bar{q}q\rangle\langle g\bar{q}\sigma\cdot Gq\rangle}(s) \\
&+ \rho_i^{\langle g\bar{q}\sigma\cdot Gq\rangle\langle g\bar{q}\sigma\cdot Gq\rangle}(s) + \rho_i^{\langle\bar{q}q\rangle\langle g^2G^2\rangle}(s) + \rho_i^{\langle\bar{q}q\rangle\langle g^3G^3\rangle}(s) + \rho_i^{\langle g^2G^2\rangle\langle g\bar{q}\sigma\cdot Gq\rangle}(s), \quad i=1,2. \qquad (13)
\end{aligned}$$

As a matter of fact, many terms of $\rho_1(s)$ are approximate to zero because they are proportional to light quarks' masses in the calculations. Thereby, we merely present the spectral densities resulted from $\Pi_2(q^2)$ here. Concretely, they can be written as

$$\begin{aligned}
\rho_2^{\text{pert}}(s) &= \frac{1}{3\cdot 5^2\cdot 2^{16}\pi^8}\int_\Lambda^1 d\alpha\frac{(1-\alpha)^6}{\alpha^5}(\alpha s-m_Q^2)^4(\alpha s+4m_Q^2), \\
\rho_2^{\langle\bar{q}q\rangle}(s) &= \frac{m_Q\langle\bar{q}q\rangle}{3\cdot 2^{11}\pi^6}\int_\Lambda^1 d\alpha\frac{(1-\alpha)^4}{\alpha^3}(\alpha s-m_Q^2)^3, \\
\rho_2^{\langle\bar{q}q\rangle^2}(s) &= \frac{\langle\bar{q}q\rangle^2}{3\cdot 2^8\pi^4}\int_\Lambda^1 d\alpha\frac{(1-\alpha)^3}{\alpha^2}(\alpha s-m_Q^2)(\alpha s+m_Q^2), \\
\rho_2^{\langle g\bar{q}\sigma\cdot Gq\rangle}(s) &= -\frac{m_Q\langle g\bar{q}\sigma\cdot Gq\rangle}{2^{11}\pi^6}\int_\Lambda^1 d\alpha\frac{(1-\alpha)^3}{\alpha^2}(\alpha s-m_Q^2)^2, \\
\rho_2^{\langle g^2G^2\rangle}(s) &= \frac{m_Q^2\langle g^2G^2\rangle}{5\cdot 3^2\cdot 2^{16}\pi^8}\int_\Lambda^1 d\alpha\frac{(1-\alpha)^6}{\alpha^5}(\alpha s-m_Q^2)(\alpha s-2m_Q^2), \\
\rho_2^{\langle g^3G^3\rangle}(s) &= \frac{\langle g^3G^3\rangle}{5\cdot 3^2\cdot 2^{18}\pi^8}\int_\Lambda^1 d\alpha\frac{(1-\alpha)^6}{\alpha^5}[(\alpha s)^2-9\alpha sm_Q^2+10m_Q^4],
\end{aligned}$$

$$\rho_2^{\langle\bar{q}q\rangle^3}(s) = \frac{m_Q\langle\bar{q}q\rangle^3}{3\cdot 2^4\pi^2}\int_\Lambda^1 d\alpha(1-\alpha),$$

$$\rho_2^{\langle\bar{q}q\rangle\langle g\bar{q}\sigma\cdot Gq\rangle}(s) = -\frac{\langle\bar{q}q\rangle\langle g\bar{q}\sigma\cdot Gq\rangle}{2^8\pi^4}s\int_\Lambda^1 d\alpha(1-\alpha)^2,$$

$$\rho_2^{\langle g\bar{q}\sigma\cdot Gq\rangle\langle g\bar{q}\sigma\cdot Gq\rangle}(s) = \frac{\langle g\bar{q}\sigma\cdot Gq\rangle^2}{2^{10}\pi^4}s\int_\Lambda^1 d\alpha(1-\alpha),$$

$$\rho_2^{\langle\bar{q}q\rangle\langle g^2G^2\rangle}(s) = \frac{m_Q\langle\bar{q}q\rangle\langle g^2G^2\rangle}{3^2\cdot 2^{13}\pi^6}\int_\Lambda^1 d\alpha\frac{(1-\alpha)^2}{\alpha^3}[6\alpha^2(\alpha s-m_Q^2)+(1-\alpha)^2(3\alpha s-4m_Q^2)],$$

$$\rho_2^{\langle\bar{q}q\rangle\langle g^3G^3\rangle}(s) = -\frac{m_Q\langle\bar{q}q\rangle\langle g^3G^3\rangle}{3\cdot 2^{14}\pi^6}\int_\Lambda^1 d\alpha\frac{(1-\alpha)^4}{\alpha^3},$$

$$\rho_2^{\langle g^2G^2\rangle\langle g\bar{q}\sigma\cdot Gq\rangle}(s) = -\frac{m_Q\langle g^2G^2\rangle\langle g\bar{q}\sigma\cdot Gq\rangle}{3\cdot 2^{13}\pi^6}\int_\Lambda^1 d\alpha\frac{(1-\alpha)^3}{\alpha^2},$$

and

$$\begin{aligned}
\hat{B}\Pi_2^{\rm cond} = & -\frac{m_Q\langle\bar{q}q\rangle^2\langle g\bar{q}\sigma\cdot Gq\rangle}{2^6\pi^2}\int_0^1 d\alpha e^{-m_Q^2/(\alpha M^2)}+\frac{m_Q^2\langle g\bar{q}\sigma\cdot Gq\rangle^2}{2^{10}\pi^4}\int_0^1 d\alpha\frac{1-\alpha}{\alpha}e^{-m_Q^2/(\alpha M^2)}\\
&+\frac{m_Q\langle\bar{q}q\rangle\langle g^2G^2\rangle^2}{3^3\cdot 2^{15}\pi^6}\int_0^1 d\alpha\frac{(1-\alpha)^2}{\alpha}\left(\frac{3}{\alpha}-\frac{m_Q^2}{\alpha^2M^2}\right)e^{-m_Q^2/(\alpha M^2)}\\
&+\frac{m_Q^3\langle\bar{q}q\rangle\langle g^3G^3\rangle}{3^2\cdot 2^{14}\pi^6}\int_0^1 d\alpha\frac{(1-\alpha)^4}{\alpha^4}e^{-m_Q^2/(\alpha M^2)}+\frac{m_Q^2\langle\bar{q}q\rangle^2\langle g^2G^2\rangle}{3^3\cdot 2^{10}\pi^4}\int_0^1 d\alpha\frac{(1-\alpha)^3}{\alpha^2}\left(\frac{2}{\alpha}-\frac{m_Q^2}{\alpha^2M^2}\right)e^{-m_Q^2/(\alpha M^2)}\\
&+\frac{\langle\bar{q}q\rangle^2\langle g^3G^3\rangle}{3^3\cdot 2^{13}\pi^4}\int_0^1 d\alpha\frac{(1-\alpha)^3}{\alpha^2}\left(\frac{11m_Q^2}{\alpha}-\frac{8m_Q^4}{\alpha^2M^2}\right)e^{-m_Q^2/(\alpha M^2)}\\
&+\frac{\langle\bar{q}q\rangle\langle g^2G^2\rangle\langle g\bar{q}\sigma\cdot Gq\rangle}{3^2\cdot 2^{11}\pi^4}\int_0^1 d\alpha\left[-1-\frac{(2-4\alpha+3\alpha^2)m_Q^2}{\alpha^3M^2}+\frac{(1-\alpha)^2m_Q^4}{\alpha^4(M^2)^2}\right]e^{-m_Q^2/(\alpha M^2)}\\
&+\frac{m_Q^3\langle g^2G^2\rangle\langle g\bar{q}\sigma\cdot Gq\rangle}{3^2\cdot 2^{13}\pi^6}\int_0^1 d\alpha\frac{(1-\alpha)^3}{\alpha^3}e^{-m_Q^2/(\alpha M^2)}\\
&+\frac{m_Q\langle g\bar{q}\sigma\cdot Gq\rangle\langle g^3G^3\rangle}{3^2\cdot 2^{14}\pi^6}\int_0^1 d\alpha\frac{(1-\alpha)^3}{\alpha^2}\left(\frac{3}{\alpha}-\frac{m_Q^2}{\alpha^2M^2}\right)e^{-m_Q^2/(\alpha M^2)} \qquad (14)
\end{aligned}$$

for $D_0^*(2400)N$ or B_0^*N state. The lower limit of integration is given by $\Lambda = m_Q^2/s$.

III. NUMERICAL ANALYSIS AND DISCUSSIONS

In this section, the sum rule (12) is numerically analyzed. The input values are taken as $m_c = 1.23\pm 0.05$ GeV, $m_b = 4.24\pm0.06$ GeV, $\langle\bar{q}q\rangle = -(0.23\pm0.03)^3$ GeV^3, $\langle g\bar{q}\sigma\cdot Gq\rangle = m_0^2\,\langle\bar{q}q\rangle$, $m_0^2 = 0.8\pm0.1$ GeV^2, $\langle g^2G^2\rangle = 0.88$ GeV^4, and $\langle g^3G^3\rangle = 0.045$ GeV^6 [21]. In the standard QCD sum rule approach, one can analyse the convergence in the OPE side and the pole contribution dominance in the phenomenological side to determine the conventional Borel window: on one hand, the lower constraint for M^2 is obtained by the consideration that the perturbative contribution should be larger than condensate contributions; on the other hand, the upper bound for M^2 is obtained by the restriction that the pole contribution should be larger than the continuum state contributions. Meanwhile, the threshold $\sqrt{s_0}$ is not arbitrary but characterizes the beginning of continuum states.

On the choice of Borel windows in this work, we would make some more discussions below. It has been shown in some Refs., e.g. [30, 31], that some condensate contributions are very large, making the standard OPE convergence to happen only at very large values of M^2. Hence one may not find the conventional Borel window. In fact, it has appeared the similar problem here. The comparison between pole and continuum contributions from sum rule (10) for $D_0^*(2400)N$ state for $\sqrt{s_0} = 3.8$ GeV is shown in the left panel of FIG. 1, and its OPE convergence by comparing the perturbative with other condensate contributions is shown in

the right panel. Even if we choose some uncritical convergence criteria, e.g. the perturbative contribution should be at least bigger than each condensate contribution, there is no standard OPE convergence up to very large values of M^2. The consequence is that it is unable to find the conventional Borel window where both the OPE converges well (i.e. the perturbative contribution bigger than each condensate contribution) and the pole dominates over the continuum.

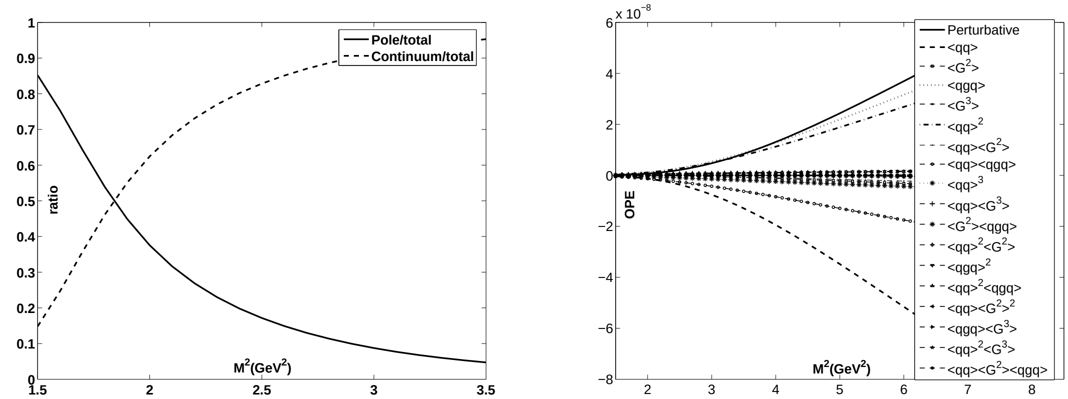

FIG. 1: In the left panel, the solid line shows the relative pole contribution (the pole contribution divided by the total, pole plus continuum contribution) and the dashed line shows the relative continuum contribution from sum rule (10) for $\sqrt{s_0} = 3.8$ GeV for $D_0^*(2400)N$ state. The OPE convergence is shown by comparing the perturbative with other condensate contributions from sum rule (10) for $\sqrt{s_0} = 3.8$ GeV for $D_0^*(2400)N$ state in the right panel.

In order to obtain some useful hadronic information from QCD sum rules, one may take into account several possible ways to solve the problem. Firstly, one could try releasing the criterion of pole dominating over continuum and take some high values of Borel parameter M^2. Thus, OPE series may converge well. However, there may occur some other problem. As one knows, the phenomenological side of the sum rule can be expressed as e.q. (4) due to the "single-pole+continuum states" hypothesis. Once the value of of M^2 is chosen too large, the single-pole dominance condition may be spoiled. Secondly, one could try pushing the threshold parameter $\sqrt{s_0}$ to a very large value, and the maximum value of M^2 will be enhanced with the increasing of $\sqrt{s_0}$. Thus, one may find the Borel window satisfying both the perturbative bigger than condensate contributions and the pole bigger than continuum contributions. However, the threshold parameter $\sqrt{s_0}$ is not arbitrary but characterizes the beginning of the continuum states. With too large values of $\sqrt{s_0}$, contributions from high resonance states and continuum states may be included in the pole contribution. Hence, the QCD sum rule may not work normally. Thirdly, one could try releasing the strict convergence criterion of perturbative contribution larger than each condensate contribution in some case. Here, we consider the ratio of perturbative contribution to the "total OPE contribution" (the sum of perturbative and other condensate contributions calculated) but not the ratio of perturbative contribution to each condensate contribution. Not too bad, there are denumerable important condensate contributions and some of them could cancel with each other to some extent since they have different signs. Most of other high dimension condensate contributions are very small. All these factors bring that the OPE convergence is still under control at relatively low values of M^2. What's more important, we find that the ratio of perturbative contribution to the "total OPE contribution" does not change much even including some condensate contributions higher than dimension 12. From the Borel curves for the $D_0^*(2400)N$ state shown in FIG. 2, one can visually see there indeed exist very stable plateaus and one could expect the OPE convergence is under control. Thus, we choose some transition range $2.0 \sim 3.0$ GeV^2 as a compromise Borel window and take the continuum thresholds as $\sqrt{s_0} = 3.7 \sim 3.9$ GeV, and arrive at 3.18 ± 0.41 GeV for $D_0^*(2400)N$ state. Considering the uncertainty rooting in the variation of quark masses and condensates,

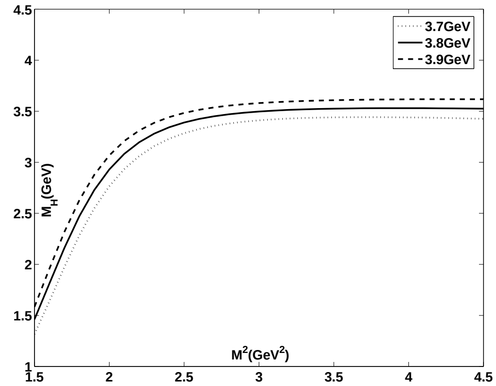

FIG. 2: The mass of $D_0^*(2400)N$ state as a function of M^2 from sum rule (12) is shown. The continuum thresholds are taken as $\sqrt{s_0} = 3.7 \sim 3.9$ GeV.

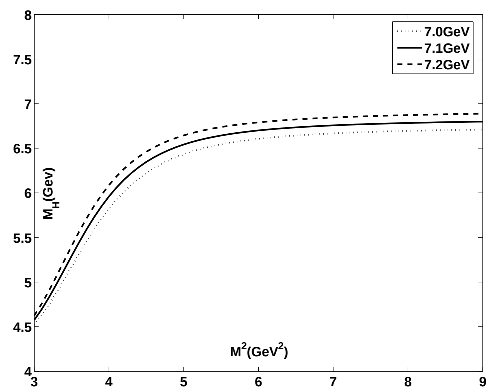

FIG. 3: The mass of B_0^*N state as a function of M^2 from sum rule (12) is shown. The continuum thresholds are taken as $\sqrt{s_0} = 7.0 \sim 7.2$ GeV.

we gain $3.18 \pm 0.41 \pm 0.10$ GeV (the first error reflects the uncertainty due to variation of $\sqrt{s_0}$ and M^2, and the second error resulted from the variation of QCD parameters) or 3.18 ± 0.51 GeV for $D_0^*(2400)N$ state. There comes forth the same problem for B_0^*N as the above case for $D_0^*(2400)N$, and we treat it similarly. The mass of B_0^*N state as a function of M^2 from sum rule (12) is shown in FIG. 3. Graphically, one can see there have very stable plateaus for Borel curves. We choose a compromise Borel window $4.5 \sim 6.0$ GeV^2 and take $\sqrt{s_0} = 7.0 \sim 7.2$ GeV for B_0^*N state. In the work windows, we obtain 6.50 ± 0.29 GeV for B_0^*N state. Varying input values of quark masses and condensates, we attain $6.50 \pm 0.29 \pm 0.20$ GeV (the first error reflects the uncertainty due to variation of $\sqrt{s_0}$ and M^2, and the second error resulted from the variation of QCD parameters) or 6.50 ± 0.49 GeV for B_0^*N state.

IV. SUMMARY AND OUTLOOK

Assuming the newly observed structure $X_c(3250)$ by BaBar Collaboration as a $D_0^*(2400)N$ molecular state, we calculate its mass value in the framework of QCD sum rules. Technically, contributions of operators up to dimension 12 are included in the operator product expansion. The final numerical result for the $D_0^*(2400)N$ state is 3.18 ± 0.51 GeV, which coincides with the experimental value 3.25 GeV. This

consolidates the statement that $X_c(3250)$ could be a $D_0^*(2400)N$ molecular state. Additionally, we have also studied the bottom counterpart B_0^*N state and predicted its mass to be 6.50 ± 0.49 GeV. By analogy with $D_0^*(2400)N$ state, this bottom counterpart state could be searched in the $\Sigma_b\pi^-\pi^-$ invariant mass spectrum in future experiments.

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